ANALYTICAL STUDY OF TURBULENT BOUNDARY LAYER DEVELOPMENT ON ROUGH SURFACE WITH ADVERSE PRESSURE GRADIENT

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ABSTRACT

One of the purposes of this investigation is to throw some additional light on the question of how to correlate between the experimental data from different sources and the theoretical approaches for the study of incompressible turbulent boundary layers development on rough surface. Moreover, the study developed included the presence of adverse pressure gradient; as well as a discussion about the effect of surface roughness on the boundary layer parameters which control the flow behaviour.

On the basis of Zancow experimental work with rough surface and the velocity law of Rotta, this study has been developed.

In order to obtain the numerical results for the mathematical solution outlined in this work, a computer program is constructed to perform all the necessary calculations. These results are represented in charts and can be utilised for the geometric shape of roughness.

NOMENCLATURE:

Symbols

A	velocity profile parameter		
B	constant		
c_∞	free stream velocity		(m/s)
$\bar{c}$	velocity at the outer edge of the boundary layer		(m/s)
c_τ	friction velocity	$\sqrt{\tau_w/\rho}$	(m/s)
$c_\tau k/\nu$	dimensionless roughness height		
$c_\tau y/\nu$	dimensionless distance normal to the wall		
c_f	local skin friction coefficient,	$\tau_w/\ 1/2\rho\bar{c}^2$	
c_x	velocity of the fluid inside the boundary layer in x-direction		(m/s)
c_y	velocity component inside the boundary layer in y-direction		(m/s)
$C(\frac{c_\tau k}{\nu})$	surface roughness function		
C_r	constant ot surface roughness function		
H_{12}	boundary layer form parameter ,	δ^* / δ^{**}	
I	boundary layer shape parameter ,	$\int_0^\infty (\frac{\bar{c}-c_x}{c_\tau})^2\, d(\frac{y\, c_\tau}{\delta^*\, \bar{c}})$	
k	roughness height		(m)
M	Mach number		
r	axi-symmetric radius		(m)
$Re_{\delta^{**}}$	momentum thickness Reynolds number ,	$\frac{\bar{c}.\delta^{**}}{\nu}$	

x	coordinate in the direction of the wall		(m)
y	coordinate normal to the direction of the wall		(m)
δ	boundary layer thickness		(m)
δ^*	boundary layer displacement thickness	, $\int_0^\infty (1-\frac{c_x}{\bar{c}})\,dy$	(m)
δ^{**}	boundary layer momentum thickness	, $\int_0^\infty \frac{c_x}{\bar{c}}(1-\frac{c_x}{\bar{c}})\,dy$	(m)
$\varkappa$	von Kármán's universal constant		
Λ	Euler number	, $-\frac{1}{\bar{c}}\cdot\frac{d\bar{c}}{dx}\cdot\delta^{**}$	
ν	kinematic viscosity of fluid		(m^2/s)
π	pressure gradient parameter	, $-\frac{1}{\bar{c}}\cdot\frac{d\bar{c}}{dx}\cdot\frac{\delta^*}{\tau_w/\rho\bar{c}^2}$	
ρ	density of fluid		(kg/m^3)
τ_w	wall shear stress		(N/m^2)

1- INTRODUCTION

In recent years, there have been increasing attempts to obtain better understanding of the behaviour of the turbulent boundary layer of incompressible flow in the presence of surface roughness. Measurements have been made in many experimental configurations and many prediction methods have been developed to get better understanding of the exact mechanism of turbulent motion, and to introduce a complete theoretical solution for that mechanism. These experimental and prediction methods included not only the effect of roughness but also the height of roughness elements, the size distribution, shape of roughness elements, and their density distribution over the wall surface.

The basis of such study depends on the solution of von Kármán momentum integral equation of boundary layers given by :

$$\frac{d\delta^{**}}{dx} + (2 + H_{12})\cdot\frac{1}{\bar{c}}\cdot\frac{d\bar{c}}{dx}\cdot\delta^{**} + \frac{\delta^{**}}{r}\cdot\frac{dr}{dx} - \frac{M^2}{\bar{c}}\cdot\frac{d\bar{c}}{dx}\cdot\delta^{**} = \frac{\tau_w}{\rho\bar{c}^2} \quad \ldots (a)$$

which is valid for laminar and turbulent flow, compressible and incompressible flow, also for plane and axi-symmetrical arrangement. For simplicity, compressibility can be neglected, if the Mach number is small compared with unity, or, in other words, if the flow velocity is small compared with the velocity of sound [1] . Therefore, the last term in the left hand side of Eq. (a) is neglected, and the third one depends on the geometry of the flow shape and can be evaluated numerically .

The remaining dimensionless relations: the wall shear stress $\tau_w / \rho.\bar{c}^2$, the form parameter H_{12} and the momentum gradient $d\delta^{**}/dx$ are functions of Euler number $\Lambda = -1/\bar{c}\,.\,d\bar{c}/dx\,.\,\delta^{**}$, and the momentum thickness Reynolds number $Re_{\delta^{**}} = \bar{c}.\,\delta^{**}/\nu$. Thus, they are given as :

$$\frac{\tau_w}{\rho\bar{c}^2} = F_1(\Lambda, Re_{\delta^{**}}) \quad \ldots (b)$$

$$H_{12} = F_2(\Lambda, Re_{\delta^{**}}) \quad \ldots (c)$$

and

$$\frac{d\delta^{**}}{dx} = F_3(\Lambda, Re_{\delta^{**}}) \quad \ldots (d)$$

Rotta [2] carried out a research from which the relationships: local skin friction coefficient $c_f = 2\tau_w/\rho\bar{c}^2$ as well as the form parameter H_{12} and Reynolds number $Re_{\delta^{**}}$ are obtained and plotted for the case of smooth surface as:

$$c_f = F_4 (Re_{\delta^{**}} , H_{12}) \qquad \ldots (e)$$

while for flow over rough surface he expressed the local skin friction coefficient in the form:

$$c_f = F_5 (Re_{\delta^{**}} , H_{12} , k/\delta^{**}) \qquad \ldots (f)$$

As the last parameter in equation (f), $\frac{k}{\delta^{**}}$, increases by increasing k , the influence of $Re_{\delta^{**}}$ decreases until it diminishes, and relation (f) yields the following form :

$$c_f = F_6 (k/\delta^{**} , H_{12}) \qquad \ldots (g)$$

Rotta plotted this relation ; including also the admissible minimum Reynolds number for rough surface ; for rough surface with sand grain roughness based on Nikuradse [3] (experimental results) with the assumption that the roughness function is not affected by the existence of pressure gradient.

Similar to the relations (b) to (d) , these relations can be expressed for rough surface as :

$$\frac{\tau_w}{\rho \bar{c}^2} = F_7 (\Lambda , k/\delta^{**}) \qquad \ldots (h)$$

$$H_{12} = F_8 (\Lambda , k/\delta^{**}) \qquad \ldots (i)$$

and

$$d\delta^{**}/dx = F_9 (\Lambda , k/\delta^{**}) \qquad \ldots (j)$$

These relations are the solution of the momentum integral equation in a convenient form because they include the effect of the pressure gradient in the form of Euler number. They are recently obtained experimentally by Zancow [4] who carried out his experiments on wind tunnel using a rotational symmetrical flow body (ellipsoidal shape) covered with different types of roughness (grinding corns). Moreover, these relations are investigated theoretically and represented in the form of charts in [5] .

Comparison between the results obtained from experiments of turbulent boundary layers on smooth and rough surfaces indicates that the existence of roughness increases the values of the dimensionless wall shear stress $\tau_w/\rho\bar{c}^2$ and the form parameter H_{12} .

2. GOVERNING EQUATIONS :

The most evaluated values of the different boundary layer parameters depend on the used form of the velocity distribution. The general acceptable form for this distribution is :

$$\frac{c_x}{c_\tau} = \frac{1}{\varkappa}\left(\ln\frac{c_\tau y}{\nu} + 2A\,\frac{y}{\delta}\right) + B + C\left(\frac{c_\tau k}{\nu}\right) , \qquad (2.1)$$

in which $\varkappa$ and B are two empirical constants of 0.4 and 5.2 values respectively, also, the contribution of surface roughness is presented by the last term of equation

(2.1). For smooth surface , $C(\frac{c_\tau k}{\nu}) = 0$, while for flow over completely rough surface it takes the form:

$$C(\frac{c_\tau k}{\nu}) = C_r - \frac{1}{\varkappa} . \ln \frac{c_\tau k}{\nu} \qquad \ldots (2.2)$$

where C_r is the constant of surface roughness function, which discussed separately in [6] .

The velocity distribution ; eq. (2.1); is similar to that suggested by Rotta [7] also independently by Ross and Robertson [8] with the exception that it does not contain the constant B taking $C(\frac{c_\tau k}{\nu}) = 5.2$ for smooth surface. The suggested form of eq. (2.1) has more generalization in its use than that given in [7] and [8].

The velocity distribution inside the boundary layer can be deduced from equation (2.1), which is given by:

$$\frac{c_x}{\bar{c}} = 1 + \frac{c_\tau}{\bar{c}} . [\frac{1}{\varkappa} . \ln \frac{y}{\delta} - \frac{2A}{\varkappa}(1 - \frac{y}{\delta})] \qquad \ldots (2.3)$$

The two boundary layer parameters defined in the nomenclature; the dimensionless displacement thickness δ^*/δ and the dimensionless momentum thickness δ^{**}/δ, are obtained by substituting the value of $c_x/\bar{c}$ in their formulae so that:

$$\frac{\delta^*}{\delta} = \frac{1}{\varkappa} . \sqrt{\frac{\tau_w}{\rho \bar{c}^2}} . (1+A) \qquad \ldots (2.4)$$

$$\frac{\delta^{**}}{\delta} = \frac{1}{\varkappa} . \sqrt{\frac{\tau_w}{\rho \bar{c}^2}} . (1+A) - \frac{1}{\varkappa^2} . \frac{\tau_w}{\rho \bar{c}^2} . (2+3A+\frac{4}{3} A^2) \qquad \ldots (2.5)$$

The remaining boundary layer parameters such as the form parameter H_{12} , shape parameter I , and the pressure gradient parameter π may all be represented by separate formulas. They are :

Form parameter $$H_{12} = \frac{\delta^*/\delta}{\delta^{**}/\delta} \qquad \ldots (2.6)$$

Shape parameter $$I = (1 - \frac{1}{H_{12}}) . \frac{1}{\sqrt{\tau_w/\rho \bar{c}^2}} = \frac{2 + 3A + \frac{4}{3} A^2}{\varkappa (1 + A)} \qquad \ldots (2.7)$$

Pressure gradient parameter $$\pi = \Lambda . \frac{H_{12}}{\tau_w/\rho \bar{c}^2} \qquad \ldots (2.8)$$

For flow past a flat plate at zero incidence , $\pi = 0$, also $\Lambda = 0$, while for boundary layers with adverse pressure gradient $\pi > 0$ and $\Lambda > 0$.

The effect of the momentum thickness Reynolds number $Re_{\delta^{**}}$ on the function $C(\frac{c_\tau k}{\nu})$ is represented by the relation :

$$C(\frac{c_\tau k}{\nu}) = \sqrt{\frac{2}{c_f}} - B - \frac{1}{\varkappa} [\ln Re_{\delta^{**}} - \ln (\frac{1+A}{\varkappa} . \frac{1}{H_{12}}) + 2A] \qquad \ldots (2.9)$$

which has been evaluated for different values of $Re_{\delta^{**}}$. As $C(\frac{c_\tau k}{\nu}) = 0$ for smooth surfaces , equation (2.9) gives the admissible minimum Reynolds number for rough surface $Re_{\delta^{**}})_{rough}$ as :

$$Re_{\delta^{**}})_{rough} = \frac{1+A}{\varkappa} \cdot \frac{1}{H_{12}} \cdot \exp\left[\varkappa\left(\sqrt{\frac{2}{c_f}} - B\right) - 2A\right] \quad \ldots (2.10)$$

According to the foregoing equation (2.10) when the actual Reynolds number $Re_{\delta^{**}}$ is lower than $Re_{\delta^{**}})_{rough}$ the surface is to be considered as being hydraulically smooth.

3- RESULTS and DISCUSSION:

A relation between the shape parameter,I, and the pressure gradient parameter, π, exists in the form $I = I(\pi)$ as given by Mellor and Gibson [9]. Another dependence exists between the pressure gradient parameter,π, and the velocity profile parameter, A. Fig. (1) illustrates this relation in which π increases with the increase of A . At $\pi = 0$ the curve intersects the A-axis at a value of ≈ 0.64.

The relation between the shape parameter I and the pressure gradient parameter π for ratios of k/δ^{**} = 0.03 , 0.05 , 0.10 and 0.30 is represented in Fig. (2) . The figure illustrates also a comparison between the present study and previous investigation given by Mellor and Gibson [9] , and shows a good agreement to the theoretical curve .

The influence of pressure gradient in form of Euler number, Λ, on the surface roughness function $C(\frac{c_\tau k}{\nu})$ is illustrated in Fig. (3) for values of the ratio k/δ^{**} = 0.05 & 0.30 with the momentum thickness Reynolds number, $Re_{\delta^{**}}$, as a parameter. At constant k/δ^{**} & Λ the function $C(\frac{c_\tau k}{\nu})$ increases as $Re_{\delta^{**}}$ decreases. Comparing Fig. 3a with 3b indicates that at the same $Re_{\delta^{**}}$ the function $C(\frac{c_\tau k}{\nu})$ decreases as the ratio k/δ^{**} increases.

The relation between $Re_{\delta^{**}})_{rough}$ and Euler number ,Λ, with k/δ^{**} as a parameter is presented in Fig. (4) . For a turbulent boundary layer with Euler number,Λ, and momentum thickness Reynolds number $Re_{\delta^{**}}$ the surface is to be considered as being hydraulically smooth when $Re_{\delta^{**}} < Re_{\delta^{**}})_{rough}$ given by Fig. (4) for a restricted value of the ratio k/δ^{**} .

For the same value of the ratio k/δ^{**} , $Re_{\delta^{**}})_{rough}$ increases with the increase of Euler number, Λ. Each curve can be divided into two parts. The first part, the slope of the curve is positive and approximately constant, which means the increase of $Re_{\delta^{**}})_{rough}$ with the increase of Euler number, Λ. In the second part, the slope of the curve increases which also increases with the decreasing of the ratio k/δ^{**}, that means large increase in the value of $Re_{\delta^{**}})_{rough}$ for small increase in Euler number, Λ.

It is also observed from the figure that as the ratio k/δ^{**} tends to zero; i.e. smooth surface; the value of $Re_{\delta^{**}})_{rough}$ goes to infinity.

4- CONCLUSIONS :

On the basis of the theoretical analysis described, and the application of the experimental results of Zancow, a number of conclusions are obtained :

The velocity profile parameter, A, increases with the increase of the pressure gradient parameter, π.

The variation of the shape parameter, I, with the pressure gradient parameter, π, for different ratios of k/δ^{**}, is in good agreement with the investigation of Mellor-Gibson.

The surface roughness function, $C\left(\frac{c_\tau k}{\nu}\right)$, for boundary layers at the same ratio of k/δ^{**} and the momentum thickness Reynolds number, $Re_{\delta^{**}}$, increases with the increase of Euler number, Λ.

The admissible minimum Reynolds number for rough surface $Re_{\delta^{**}})_{rough}$ increases with the increase of Euler number, Λ, and the decrease of the ratio k/δ^{**}.

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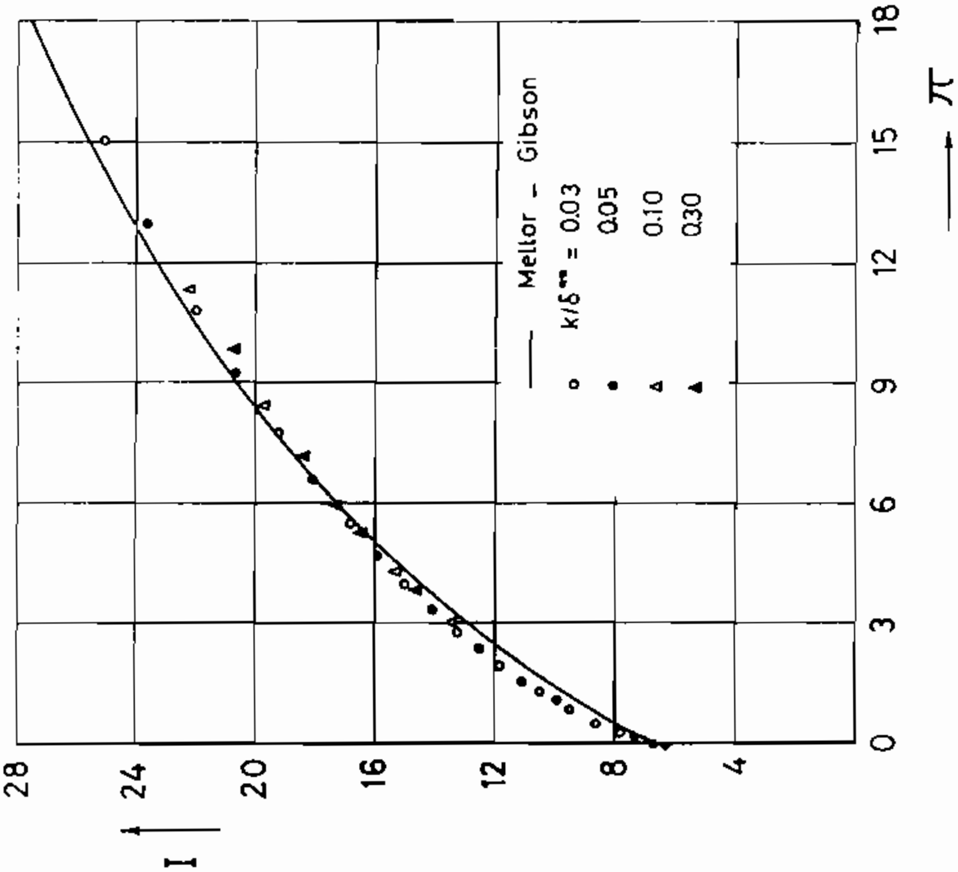

Fig.(2) Shape parameter versus pressure gradient parameter

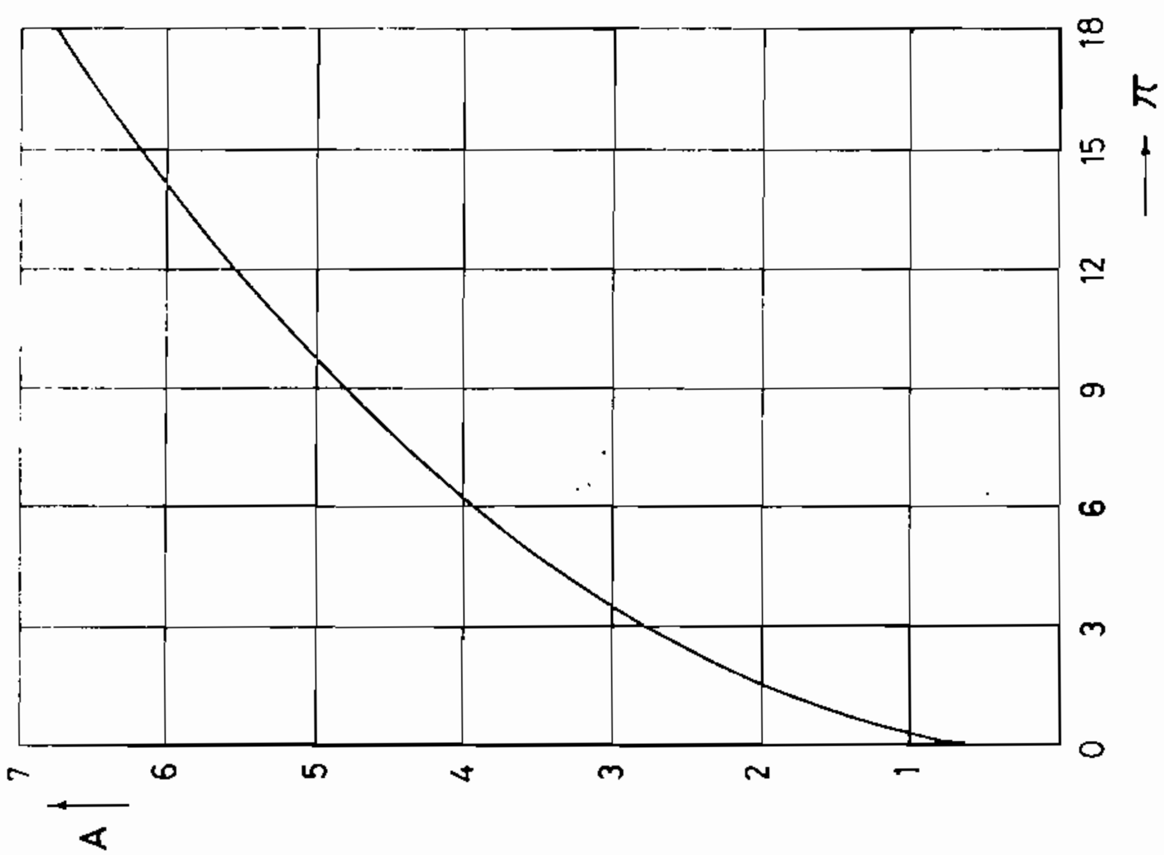

Fig.(1) Velocity profile parameter versus pressure gradient parameter

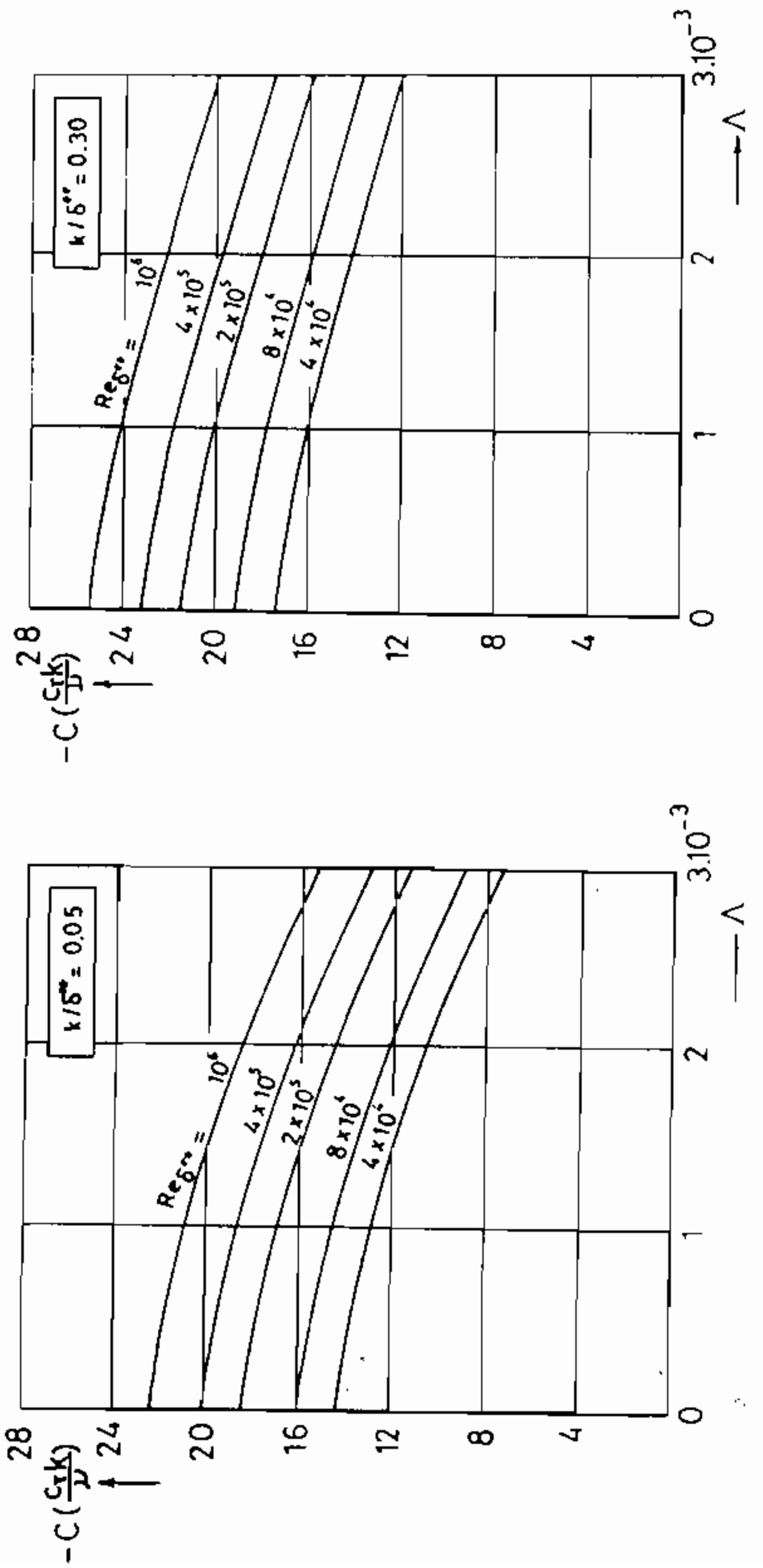

Fig.(3) Surface roughness function versus Euler number with momentum thickness Reynolds number as parameter, at constant ratios of k/δ^{**}

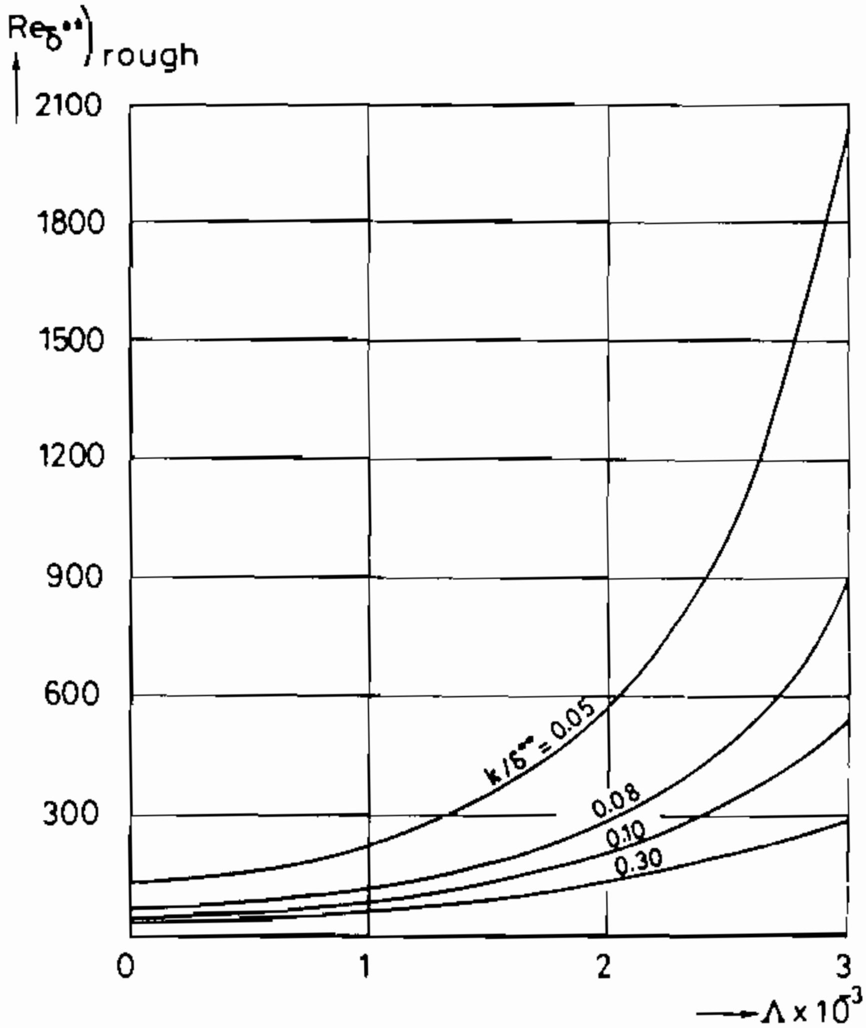

Fig. (4) Admissible minimum Reynolds number for rough surface versus Euler number with the ratio k/δ^{**} as parameter